Disk Packing for the Estimation of the Size of a Wire Bundle

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Abstract

A heuristic method for packing disks in a circle is constructed, and is applied to the estimation of the sizes of holes through which given sets of electric wires are to pass. In the proposed method, a sufficiently large circle is initially constructed, and it is shrunk step by step while keeping all the disks inside. For this purpose a Voronoi diagram for circles is used. Computational experiments show the validity and the efficiency of the method.

Key words: circle packing, circle Voronoi diagram, wire bundling, bundle size, automobile industry

1. Introduction

Modern intelligent cars contain many electric systems. For example, a car has many sensors such as temperature sensors and speed sensors, many controllers such as engine controllers and break controllers, many power electricity such as an engine starter and a power steering, many lights such as head lights and break lamps, and many additional systems such as a global positioning system, a CD player, sound speakers and a radio. Consequently a large number of electric wires run in the body of a car in a complicated manner.

In a bundle of electric wires used in an actual car, a set of wires of various sizes are collected together into a cylinder-like bundle, and many bundles are again collected together into a larger bundle. In order to layout those bundles of wires, we have to make holes on walls of the body through which the wires pass. Those holes should be large enough for the bundle of wires to pass, but we do not want to make them unnecessarily large because larger holes will weaken the body.

Hence, we want to estimate the size of a bundle of wires as precisely as possible in order to determine the sizes of the holes. From the precision point of view the best way is to actually make a bundle using real wires. However we do not want to do that, because we want to know the appropriate sizes of the holes in the design stage, where everything is just on sheets of paper or in computers. Thus, given a set of the sizes of wires, we want to estimate the size of the bundle that would be generated when we put them together tightly. This is the problem we consider in this paper.

Considering the cross section of a wire bundle, we may reduce the problem into a two-dimensional disk packing problem, that is, given a set of disks (corresponding to the cross sections of wires), we want to find the smallest enclosing circle that encloses all the disks without overlap.

Disk packing is an old and hard problem, and has been studied by many mathematicians. Examples of old work can be found in Goldberg [2] and Kravitz [3]. Since these days, various types of packing problems have been studied. They include packing disks in a circle [4], packing disks in a square [5], and packing disks on a sphere [2]. They search for the smallest size of the enclosing figure in a strict sense, and gave solutions only to some restricted numbers of disks. Moreover, they concentrate their attention onto the case where all the disks are of the same size. Actually there is almost no result for disks with different sizes.

Fortunately, however, what we want in order to estimate the size of a wire bundle is not the strictly smallest enclosing circle, but an approximation that is nearly smallest. Such enclosing circles were studied by Drezner and Erkut [1]; they reduced the problem to a nonlinear optimization problem, but they reported that their method was time consuming and could compute the circles enclosing up to 23 disks. What we want, on the other hand, is an enclosing circle that encloses several hundreds of disks with different sizes, and hence we cannot use the previous methods.

In this paper, we propose a new method for finding a small circle that encloses a given set of disks with different sizes. Intuitively speaking, this method simulates a physical process in which we shake the disks inside the enclosing circle while we shrink the enclosing circle step by step until the disks cannot move any more.

2. Bundle Coefficient Approach

Let us denote by c_i a disk with radius r_i. The location of c_i is not fixed; we can move c_i in the plane freely. Suppose that we are given a set $C = \{c_1, c_2, \cdots, c_n\}$ of n such disks. Our goal is to find a nearly smallest circle that encloses all the disks without overlap in reasonably small computation time.

Table 1 shows an example a disk set that actually arises in an existing car. In this table, the left column shows the category names of wires, the middle column shows the radii of the perpendicular sections of the wires, and the right column shows the number of wires belonging to each category and each size. Hence, in this example, the bundle contains 162 wires, among which the largest one is more than three times larger in radius than the smallest one.

Table 1. Wires belonging to a bundle in an actual car.

category of a wire	radius (mm)	number
23A	1.8	3
20G	0.7	11
31G	0.8	31
44G	0.9	5
45G	1.05	9
51G	1.3	12
25J	0.55	25
28J	0.65	54
30J	0.75	1
31J	0.9	6
22X	0.9	4
10E	1.75	1
total		162

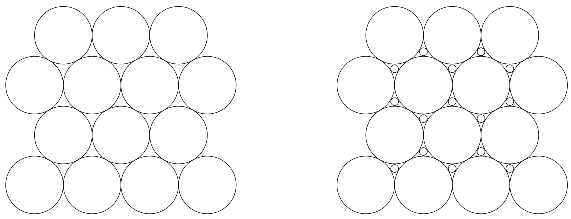
We first describe what is currently being done in industry. They have a secret number, say B, called a "bundle coefficient". Let S be the sum of the areas of the disks:

$$S = \sum_{i=1}^{n} \pi {r_i}^2.$$

Then, they estimate the diameter of the bundle by $B\sqrt{S}$.

This estimation is based on the assumption that in the section of a wire bundle, the ratio of the area occupied by the wires to the empty area is constant.

However, this is not true in general. A typical example is shown in Fig. 1. Fig. 1(a) shows the case where all the disks are equal in size, and they are packed tightly in the way just like a nest of bees. In Fig. 1(b), on the other hand, additional smaller disks are inserted in the empty space. Therefore, the ratio of the occupied area is larger in Fig. 1(b) than in Fig. 1(a). This example implies that the bundle-coefficient method does not work well, because the distribution of wire sizes changes from bundle to bundle.



(a) (b)

Figure 1. Two packings in which the ratio of the empty space is different.

Actually there are more than one bundle coefficient in industry, and experienced persons select according to their inspiration the one that seems the most suitable for each set of wires. This kind of estimation cannot guarantee the quality of the result, and hence a more logical method is required.

3. Shrink-and-Shake Algorithm

In this section we present the basic structure of the proposed algorithm. First, we show an example of the behavior of our algorithm, next sketch the basic structure, and then describe how to simulate shaking disks in a circle.

4. Concluding Remarks

In this paper, we reduced the problem of estimating the size of a wire bundle to the two-dimensional disk packing problem, and proposed a shrink-and-shake method for solving this problem (Algorithm 1). The proposed method is heuristic in its nature, but the experiments show that the output is stable in the sense that the variance of the estimated size of the enclosing circle is very small. Moreover, the size of the enclosing circle obtained by our algorithm is close to the actual size of the wire bundle; it is actually close enough for our original goal of estimating the size of the hole we should make.

From the time complexity point of view, we presented two methods, a naive method (Algorithm 2) and an improved method (Algorithm 3), for the main part of the shrink-and-shake procedure. Experiments show that the improved method gives better performance for large numbers of disks. Also we found that the naive method is useful to solve moderate sizes of problems arising in actual industry.

Our next question is how large the number of disks for the improved method to show a better performance than the naive method in the case where the sizes of disks are different. This is one of our future problems.

There are many related problems for future. In order to make a better estimation of the size of a wire bundle, we need to consider the friction of insulator and the three-dimensional structure of wires.

In this paper we presented the shrink-and-shake strategy, but other strategies might be also possible. For example, when we push the protruding disk toward inside the enclosing circle, the force propagates from disk to disk. This physical phenomenon can also be formulated in a mathematical manner in order to get another strategy.

As for the disk packing problem, we can consider many variants. For example, other problems are obtained when we replace the enclosing circle with other shapes, such as an enclosing square, an enclosing rectangle, an enclosing triangle and an enclosing ellipse. It might be also a challenge to consider a non-convex enclosing shape.

References

[1] Z. Drezner and E. Erkut: Solving the continuous p-dispersion problem using non-linear programming. *Journal of the Operations Research Society*, vol. 46 (1995), pp. 516–520.

[2] M. Goldberg: Packing of 18 equal circles on a sphere. *Elements of Mathematics*, vol. 20 (1965), pp. 59–61.

[3] S. Kravitz: Packing cylinders into cylindrical containers. *Mathematical Magazine*, vol. 40 (1967), pp. 65–71.

[4] B. D. Lubachevsky and R. L. Graham: Curved hexagonal packings of equal disks in a circle. *Discrete and Computational Geometry*, vol. 18 (1997), pp. 179–194.

[5] M. Mollard and C. Payan: Some progress in the packing of equal circles in a square. *Discrete Mathematics*, vol. 84 (1990), pp. 303–307.